AP® Calculus BC

Mr. Ewell’s Syllabus

Limits and Their Properties
• An introduction to limits, including an intuitive understanding of the limit process
• Using graphs and tables of data to determine limits
• Properties of limits
• Algebraic techniques for evaluating limits
• Comparing relative magnitudes of functions and their rates of change
• Continuity and one-sided limits
• Geometric understanding of the graphs of continuous functions
• Intermediate Value Theorem
• Infinite limits
• Using limits to find the asymptotes of a function

Differentiation
• Zooming-in activity and local linearity
• Understanding of the derivative: graphically, numerically, and analytically
• Approximating rates of change from graphs and tables of data
• The derivative as: the limit of the average rate of change, an instantaneous rate of change, limit of the difference quotient, and the slope of a curve at a point
• The meaning of the derivative—translating verbal descriptions into equations and vice versa
• The relationship between differentiability and continuity
• Functions that have a vertical tangent at a point
• Functions that have a point on which there is no tangent
• Differentiation rules for basic functions, including power functions and trigonometric functions
• Rules of differentiation for sums, differences, products, and quotients
• The chain rule
• Implicit differentiation
• Related rates

Applications of Differentiation
• Extrema on an interval and the Extreme Value Theorem
• Rolle’s Theorem and the Mean Value Theorem, and their geometric consequences
• Increasing and decreasing functions and the First Derivative Test
• Concavity and its relationship to the first and second derivatives
• Second Derivative Test
• Limits at infinity
• A summary of curve sketching—using geometric and analytic information as well as calculus to predict the behavior of a function
• Relating the graphs of \( f, f', \) and \( f'' \)
• Optimization including both relative and absolute extrema
• Tangent line to a curve and linear approximations
• Application problems including position, velocity, acceleration, and rectilinear motion
Integration
- Antiderivatives and indefinite integration, including antiderivatives following directly from derivatives of basic functions
- Basic properties of the definite integral
- Area under a curve
- Meaning of the definite integral
- Definite integral as a limit of Riemann sums
- Riemann sums, including left, right, and midpoint sums
- Trapezoidal sums
- Use of Riemann sums and trapezoidal sums to approximate definite integrals of functions that are represented analytically, graphically, and by tables of data
- Use of the First Fundamental Theorem to evaluate definite integrals
- Use of substitution of variables to evaluate definite integrals
- Integration by substitution
- The Second Fundamental Theorem of Calculus and functions defined by integrals
- The Mean Value Theorem for Integrals and the average value of a function

Logarithmic, Exponential, and Other Transcendental Functions
- The natural logarithmic function and differentiation
- The natural logarithmic function and integration
- Inverse functions
- Exponential functions: differentiation and integration
- Bases other than e and applications
- Solving separable differential equations
- Applications of differential equations in modeling, including exponential growth
- Use of slope fields to interpret a differential equation geometrically
- Drawing slope fields and solution curves for differential equations
- Euler’s method as a numerical solution of a differential equation
- Inverse trig functions and differentiation
- Inverse trig functions and integration

Applications of Integration
- The integral as an accumulator of rates of change
- Area of a region between two curves
- Volume of a solid with known cross-sections
- Volume of solids of revolution
- Arc length
- Applications of integration in physical, biological, and economic contexts
- Applications of integration in problems involving a particle moving along a line, including the use of the definite integral with an initial condition and using the definite integral to find the distance traveled by a particle along a line
Integration Techniques, L’Hopital’s Rule, and Improper Integrals
• Review of basic integration rules
• Integration by parts
• Trigonometric integrals
• Integration by partial fractions
• Solving logistic differential equations and using them in modeling
• L’Hopital’s Rule and its use in determining limits
• Discovery activity on improper integrals
• Improper integrals and their convergence and divergence, including the use of L’Hopital’s Rule

Infinite Series
• Convergence and divergence of sequences
• Definition of a series as a sequence of partial sums
• Convergence of a series defined in terms of the limit of the sequence of partial sums of a series
• Introduction to convergence and divergence of a series by using technology on two examples to gain an intuitive understanding of the meaning of convergence
• Geometric series and applications
• The nth-Term Test for Divergence
• The Integral Test and its relationship to improper integrals and areas of rectangles
• Use of the Integral Test to introduce the test for p-series
• Comparisons of series
• Alternating series and the Alternating Series Remainder
• The Ratio and Root Tests
• Taylor polynomials and approximations: introduction using the graphing calculator
• Power series and radius and interval of convergence
• Taylor and Maclaurin series for a given function
• Maclaurin series for sin x, cos x, e^x and \( \frac{1}{1-x} \)
• Manipulation of series, including substitution, addition of series, multiplication of series by a constant and/or a variable, differentiation of series, integration of series, and forming a new series from a known series
• Taylor’s Theorem with the Lagrange Form of the Remainder

Plane Curves, Parametric Equations, and Polar Curves
• Plane curves and parametric equations
• Parametric equations and calculus
• Parametric equations and vectors: motion along a curve, position, velocity, acceleration, speed, distance traveled
• Analysis of curves given in parametric and vector form
• Polar coordinates and polar graphs
• Analysis of curves given in polar form
• Area of a region bounded by polar curves
1. Each topic is presented numerically, geometrically, symbolically, and verbally as students learn to communicate the connections among these representations. They have frequent opportunities to verbally discuss as a group, using precise calculus language, their interpretation of the topic at hand.

2. Justifications of responses and solutions are part of the routine when solving problems. Students are encouraged to express their ideas in carefully written sentences that validate their process and conclusions.

3. Each student has his/her own TI-89 calculator. The calculator helps students develop a visual understanding of the material that would be otherwise difficult to see.

4. Students use programs in their calculators to:
   - Graph functions over an arbitrary interval
   - Find zeros of functions
   - Find intersections of functions
   - Use zoom to investigate local linearity
   - Find derivatives
   - Perform numerical integration
   - Find points of inflection
   - Show Riemann sums
   - Compute partial sums
   - Use Euler’s method
   - Show a slope field
   - Draw a solution curve on a slope field
   - Sketch implicitly defined functions

5. Mr. Ewell does not give Summer Projects.

6. Students should expect to spend about 45 minutes per night for homework.

7. This is the only AP class that covers 2 semesters of college work. It is recommended that students make every effort to not miss classes. Doing homework is a must. The greatest predictor of success is not being specially gifted. Intellectual curiosity and the willingness to explore new concepts beyond “common sense” makes the experience a positive, rewarding, and enjoyable one.